

A remark on the paper “Redundancy of multiset topological spaces”

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Abstract

In this note the authors have raised the question regarding the validity of the main result in [1] by setting an example.

In [1] the author claimed that the multiset topology defined on a set X is equivalent to the general topology defined on the product $X \times \mathbb{N}$. But there are some doubts in this statement. In fact, the main result in [1] is the Theorem 3.4 and by the following example it is seen that in proof of Theorem 3.4 the results:

- (1) $\varphi(\sqcup M_i) = \bigcup_{i \in J} \varphi(M_i)$,
 - (2) $\varphi(\sqcup M_i) = \bigcup_{i \in J} \varphi(M_i)$ and
 - (3) $\varphi(M^\Delta) = (\varphi(M))^c$
- do not hold.

Example 1. Let $X = \{x, y, z\}$, $\omega = 4$ and $U, V \in [X]^\omega$ such that

$U = \{4/x, 3/y, 2/z\}$, $V = \{4/x, 4/y, 4/z\}$.

Let $M_1, M_2 \in P^*(U)$ so that $M_1 = \{4/x, 3/y\}$ and $M_2 = \{2/x, 3/y\}$.

Then $M_1 \sqcup M_2 = \{4/x, 3/y\}$, $M_1 \cap M_2 = \{2/x, 3/y\}$. The complement of M_2 in $[X]^\omega$ is $M_2^\Delta = \{2/x, 1/y, 4/z\}$ and the complement of M_2 with respect to U is $(M_2^\Delta)_U = \{2/x, 2/z\}$.

Therefore, $\varphi(M_1) = \{(x, 4), (y, 3)\}$, $\varphi(M_2) = \{(x, 2), (y, 3)\}$, $\varphi(M_1) \cup \varphi(M_2) = \{(x, 4), (x, 2), (y, 3)\}$, $\varphi(M_1) \cap \varphi(M_2) = \{(y, 3)\}$, $\varphi(M_1 \sqcup M_2) = \{(x, 4), (y, 3)\}$ and $\varphi(M_1 \cap M_2) = \{(x, 2), (y, 3)\}$.

Again, $\varphi(M_2^\Delta) = \{(x, 2), (y, 1), (z, 4)\}$ and $\varphi((M_2^\Delta)_U) = \{(x, 2), (z, 2)\}$.

Now, $\varphi(U) = \{(x, 4), (y, 3), (z, 2)\}$ and $\varphi(V) = \{(x, 4), (y, 4), (z, 4)\}$.

Then $\varphi(U) \setminus \varphi(M_2) = \{(x, 4), (z, 2)\}$, $\varphi(V) \setminus \varphi(M_2) = \{(x, 4), (y, 4), (z, 4)\}$,

$X \times \mathbb{N} \setminus \varphi(M_2) = X \times \mathbb{N} \setminus \{(x, 2), (y, 3)\}$ and

$(X \times \{1, 2, 3, 4\}) \setminus \varphi(M_2) = \{(x, 1), (x, 3), (x, 4), (y, 1), (y, 2), (y, 4), (z, 1), (z, 2), (z, 3), (z, 4)\}$.

From the above results it follows that none of (1), (2) and (3) in Theorem 3.4 of [1] holds.

References

- [1] A. Ghareeb, Redundancy of multiset topological spaces, arXiv: 1606.01150v1 [math.GM] 2 Jun 2016.